



# Modeling, experimental study and optimization on low-temperature waste heat thermoelectric generator system

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## ABSTRACT

Thermoelectric generation technology, due to its several kinds of merits, especially its promising applications to waste heat recovery, is becoming a noticeable research direction. Based on basic principles of thermoelectric generation technology and finite time thermodynamics, thermoelectric generator system model has been established. In order to investigate viability and further performance of the thermoelectric generator for waste heat recovery in industry area, a low-temperature waste heat thermoelectric generator setup has been constructed. Through the comparison of results between theoretic analysis and experiment, reasonability of this system model has been verified. Testing results and discussion show the promising potential of using thermoelectric generator for low-temperature waste heat recovery, especially in industrial fields. Several suggestions for system performance improvement have been proposed through the analysis on this system model, which guide optimization and modification of this experimental setup. By integrating theoretic analysis and experiment, it is found that besides increasing waste heat temperature and TE modules in series, expanding heat sink surface area in a proper range and enhancing cold-side heat transfer capacity in a proper range can also be employed to enhance performance of this setup.

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## 1. Introduction

Thermoelectric generation technology, as one entirely solid-state energy conversion way, can directly transform thermal energy into electricity by using thermoelectric transformation materials. A thermoelectric power converter has no moving parts, and is compact, quiet, highly reliable and environmentally friendly. Due to these merits, this generation technology is presently becoming a noticeable research direction [1–14].

However, wide application of thermoelectric power generation has been limited because of its relatively low heat-to-electricity conversion efficiency. Nowadays, a large number of works concerning thermoelectricity focus on how to improve heat-to-electricity transformation efficiency of thermoelectric materials, that is, how to improve the thermoelectric figure of merit  $ZT$  of these materials. Nanotechnology [3–6], novel technology in solid state physics and semiconductor physics [2,7,8] are employed to explore this exciting field.

Despite these promising results, efficiency gained at device level has yet to be demonstrated. The scaling of the nanomaterials has proven to be quite difficult and is still in its developing stage.

The bulk material has yet to be made commercially available. Bell [1] points out two important pathways that will lead to additional applications of thermoelectric (TE) devices. One is to promote the intrinsic efficiencies of TE materials. The other is to improve the way where existing TEs are currently used. The key factor should be the usage of economic and efficient heat sources.

Currently, there are a large number of waste heats in our surroundings, especially in the field of industrial production, which can not be recycled effectively by conventional methods. TE generation technology appears to have advantages in this area of low-grade waste heat recovery due to its entire solid-state energy conversion mode. Qiu and Hayden [15] develop a self-powered residential heating system using thermoelectric power generation technology. The electricity generated is adequate to power all electrical components of a residential central heating system. Furthermore, considering waste heats are low-cost and even no-cost resources, added with the benefits of energy-saving and emission reduction, the low efficiency problem is no longer the most important issue that we have to take into account. Economic viability of a TE generator may be improved significantly when used for waste heat recovery. Niu et al. [16] have recently done a study concerned.

In the aspect of system analysis and optimization, Rowe and Gao [17] develop a procedure to assess the potential of thermoelectric modules used for electrical power generation. Chen and Wu [18] use an irreversible model to study the performance of a

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thermoelectric generator with external and internal irreversibility. Esarte et al. [19] apply an NTU- $\varepsilon$  methodology to study the influence of fluid flow rate, heat exchanger geometry, fluid properties and inlet temperatures on the power supplied by thermoelectric generator. They have analyzed the effect of heat transfer between thermoelectric device and its external heat reservoirs on performance of a single-element thermoelectric generator. In practice, a thermoelectric generator is composed of many fundamental thermoelectric elements. It is a multi-element device.

In this paper, a more reasonable system model of thermoelectric generator has been adopted for system analysis and optimization [20]. Besides, characteristics of a multi-element thermoelectric generator with the irreversibility of finite rate heat transfer, Joule heat inside the thermoelectric device, and the heat leak through the thermoelectric couple leg have been investigated.

In order to investigate viability and further performance of the thermoelectric generator for waste heat recovery in industry area, a low-temperature waste heat thermoelectric generator setup has been constructed. By integrating theoretic analysis and experiment, this paper studies the influence of heat transfer irreversibility on thermoelectric generation performance, and several pieces of advices on improvement of system performance have been proposed.

Testing results and discussion show the promising potential of using thermoelectric generator for low-temperature waste heat recovery in industrial fields. The system model established can be employed in performance optimization and further application of thermoelectric generation.

## 2. System modeling

### 2.1. Mathematical model

A general thermoelectric generator with a load resistance  $R_L$  connected is composed of several thermoelectric elements (Fig. 1). Each element consists of P- and N-type semiconductor legs which work between high and low temperature heat reservoirs whose temperature are  $T_H$  and  $T_L$  respectively.

$Q_H$  and  $Q_L$  in Fig. 1 present the heat the generator absorbs from high temperature reservoir and the heat it releases to low temperature reservoir per unit time respectively, that is, the heat flux between the generator and the two heat reservoirs. Owing to Peltier effect, the heat flux each thermoelectric element of the generator (one pair of P- and N-type semiconductor legs) absorbs from high temperature reservoir ( $Q_1$ ) and releases to low temperature reservoir ( $Q_2$ ) can be shown as follows:

$$Q_1 = \alpha I T_1 \quad (1)$$

$$Q_2 = \alpha I T_2 \quad (2)$$

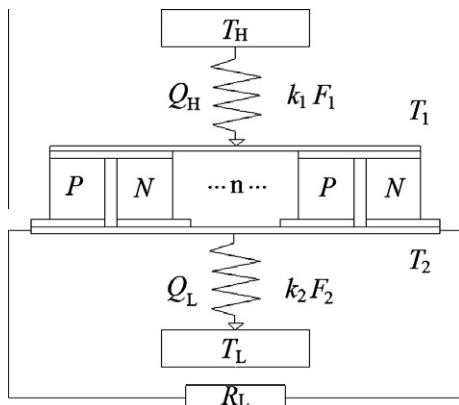


Fig. 1. Schematic diagram of multi-element thermoelectric generator.

where  $I$  is electric current in generator circuit,  $T_1$  and  $T_2$  are hot-side and cold-side temperature of the generator,  $\alpha_P$  and  $\alpha_N$  are the Seebeck coefficients of the P- and N-type semiconductor legs respectively, and  $\alpha = \alpha_P - \alpha_N$ .

Joule heat flux one pair of semiconductor legs generates when current passes through is

$$Q_J = I^2 R \quad (3)$$

where

$$R = l_P / (\sigma_P A_P) + l_N / (\sigma_N A_N) \quad (4)$$

$R$  is electric resistance of semiconductor couple, while  $l_P$ ,  $\sigma_P$ ,  $A_P$  and  $l_N$ ,  $\sigma_N$ ,  $A_N$  are length, conductivity and cross sectional area of P- and N-type legs respectively. For the purpose of facilitating the calculation, thermoelectric element is assumed to be insulated thermally, from its surroundings, except at the junction-reservoir contacts [21].

According to Newton's law of heat transfer, a heat flux is induced by the temperature difference between both ends of the thermoelectric element,

$$Q_K = K(T_1 - T_2) \quad (5)$$

The heat flux in the legs is carried from hot-side to cold-side, where,

$$K = \lambda_P A_P / l_P + \lambda_N A_N / l_N \quad (6)$$

$K$  is thermal conductance of the semiconductor couple ( $W K^{-1}$ ), while  $\lambda_P$  and  $\lambda_N$  are thermal conductivity of P- and N-type legs respectively.

On account of a heat resistance existing between reservoirs and generator, heat exchange rate is limited, namely a finite time heat transfer [18]. Similarly, by employing Newton's law of heat transfer and with analysis mentioned above,  $Q_H$  and  $Q_L$  can be expressed respectively as follows:

$$\begin{aligned} Q_H &= k_1 F_1 (T_H - T_1) = n(Q_1 + Q_K - 0.5Q_J) \\ &= n[\alpha I T_1 + K(T_1 - T_2) - 0.5I^2 R] \end{aligned} \quad (7)$$

$$\begin{aligned} Q_L &= k_2 F_2 (T_2 - T_L) = n(Q_2 + Q_K + 0.5Q_J) \\ &= n[\alpha I T_2 + K(T_1 - T_2) + 0.5I^2 R] \end{aligned} \quad (8)$$

where  $k_1$ ,  $k_2$  are heat transfer coefficients in hot-side and cold-side heat exchangers respectively ( $W m^{-2} K^{-1}$ ),  $F_1$ ,  $F_2$  are heat transfer surface areas in hot-side and cold-side heat exchangers respectively, and  $n$  is the number of thermoelectric elements.

This study focuses on low-temperature system, and the temperature difference between two reservoirs is relatively small accordingly. Therefore, the influence of Thomson effect could be ignored in this analysis [18].

### 2.2. Power and efficiency equations

Similar with the method of Chen [22], combining (7) and (8),

$$\begin{aligned} T_1 &= \left[ k_1 F_1 k_2 F_2 T_H + nK(k_1 F_1 T_H + k_2 F_2 T_L) - n\alpha k_1 F_1 T_H I \right. \\ &\quad \left. + (0.5nk_2 F_2 + n^2 K) R I^2 - 0.5\alpha R n^2 I^3 \right] \\ &\quad \times \left[ k_1 F_1 k_2 F_2 + nK(k_1 F_1 + k_2 F_2) + n\alpha(k_2 F_2 - k_1 F_1) I - n^2 \alpha^2 I^2 \right]^{-1} \end{aligned} \quad (9)$$

$$\begin{aligned} T_2 &= \left[ k_1 F_1 k_2 F_2 T_L + nK(k_1 F_1 T_H + k_2 F_2 T_L) + n\alpha k_2 F_2 T_L I \right. \\ &\quad \left. + (0.5nk_1 F_1 + n^2 K) R I^2 + 0.5\alpha R n^2 I^3 \right] \\ &\quad \times \left[ k_1 F_1 k_2 F_2 + nK(k_1 F_1 + k_2 F_2) + n\alpha(k_2 F_2 - k_1 F_1) I - n^2 \alpha^2 I^2 \right]^{-1} \end{aligned} \quad (10)$$

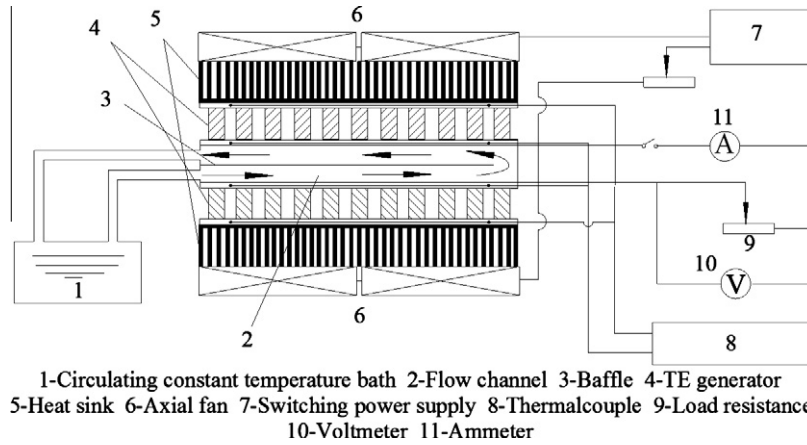


Fig. 2. Schematic diagram of the experimental thermoelectric generator setup.

$$Q_H = \left\{ k_1 F_1 \left[ 0.5 \alpha R n^2 I^3 - (0.5 n k_2 F_2 R + n^2 K R + n^2 \alpha^2 T_H) I^2 + n \alpha k_2 F_2 T_H I + n K k_2 F_2 (T_H - T_L) \right] \right\} \times \left[ k_1 F_1 k_2 F_2 + n K (k_1 F_1 + k_2 F_2) + n \alpha (k_2 F_2 - k_1 F_1) I - n^2 \alpha^2 I^2 \right]^{-1} \quad (11)$$

$$Q_L = \left\{ k_2 F_2 \left[ 0.5 \alpha R n^2 I^3 + (0.5 n k_1 F_1 R + n^2 K R + n^2 \alpha^2 T_L) I^2 + n \alpha k_1 F_1 T_L I + n K k_1 F_1 (T_H - T_L) \right] \right\} \times \left[ k_1 F_1 k_2 F_2 + n K (k_1 F_1 + k_2 F_2) + n \alpha (k_2 F_2 - k_1 F_1) I - n^2 \alpha^2 I^2 \right]^{-1} \quad (12)$$

Meanwhile, the power output  $P$  and the thermal efficiency  $\eta$  of the semiconductor thermoelectric generator are, respectively:

$$P = Q_H - Q_L = n \left[ \alpha I (T_1 - T_2) - I^2 R \right] = \left\{ 0.5 \alpha R n^2 I^3 (k_1 F_1 - k_2 F_2) - [n R k_1 F_1 k_2 F_2 + n^2 K R (k_1 F_1 + k_2 F_2) + n^2 \alpha^2 (k_1 F_1 T_H + k_2 F_2 T_L)] I^2 + n \alpha k_1 F_1 k_2 F_2 (T_H - T_L) I \right\} \times \left[ k_1 F_1 k_2 F_2 + n K (k_1 F_1 + k_2 F_2) + n \alpha (k_2 F_2 - k_1 F_1) I - n^2 \alpha^2 I^2 \right]^{-1} \quad (13)$$

$$\eta = P / Q_H = \left\{ 0.5 \alpha R n^2 I^3 (k_1 F_1 - k_2 F_2) - [n R k_1 F_1 k_2 F_2 + n^2 K R (k_1 F_1 + k_2 F_2) + n^2 \alpha^2 (k_1 F_1 T_H + k_2 F_2 T_L)] I^2 + n \alpha k_1 F_1 k_2 F_2 (T_H - T_L) I \right\} \times \left\{ k_1 F_1 \left[ 0.5 \alpha R n^2 I^3 - (0.5 n k_2 F_2 R + n^2 K R + n^2 \alpha^2 T_H) I^2 + n \alpha k_2 F_2 T_H I + n K k_2 F_2 (T_H - T_L) \right] \right\}^{-1} \quad (14)$$

Eqs. (13) and (14) reflect the relationship among power output and thermal efficiency of the generator, the irreversibility in external heat exchange, the number of thermoelectric modules, working current, and the irreversible factors internal, which are the important theoretical basis for analysis and optimization of the generator system performance.

### 3. Experimental study on low-temperature waste heat thermoelectric generator

#### 3.1. Experimental setup

We have designed a flow channel (size: 29 cm by 13 cm by 5 cm, a baffle embedded) to utilize waste heat resource when ther-

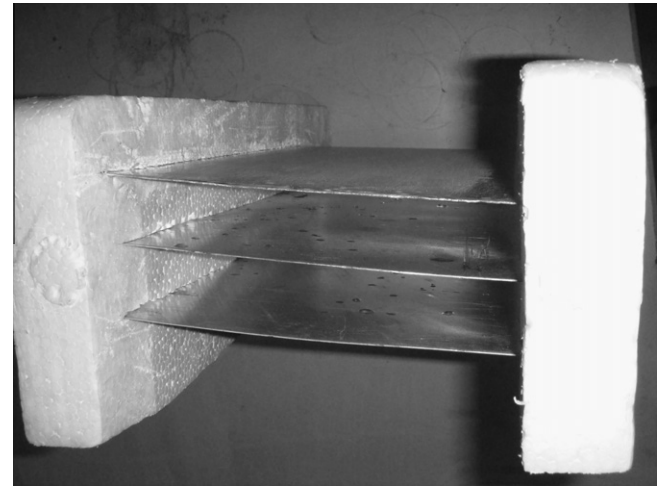


Fig. 3. Flow channel.

mal fluid flows through the channel, namely, using temperature difference between thermal fluid and ambient air to generate electricity by TE modules fixed on the flow channel surface (Fig. 2).

There are ten TE modules (Model No. TEC1-03180T125), arranged in two lines, fixed on one side of the channel surface (Fig. 3) along the fluid flowing direction. The modules are connected electrically in series, but thermally in parallel. Some measuring instruments are equipped to construct this experimental setup.

Only upper surface of the channel is paved with modules in this study, and the lower surface is reserved for further research. A circulating constant temperature water bath (temperature range: room temperature –100 °C, built-in circulating pump flow rate: 6 L min<sup>-1</sup>) is adopted as a power source to drive thermal fluid, that is, applying this circulating water bath to simulate waste heat flow. An axial fan (diameter: 12 cm, 12 V/0.55 A), together with ten heat sinks (base size: 4 cm by 4 cm), are mounted to enhance heat dissipation on cold-side of the modules.

#### 3.2. Performance testing

For this setup, hot-side temperature is relatively controllable and stable. System performance is subject to cold-side heat transfer. Thus, some test results have been attained while changing with hot water (waste heat) temperature (by controlling constant temperature bath). In order to acquire guidance on development and

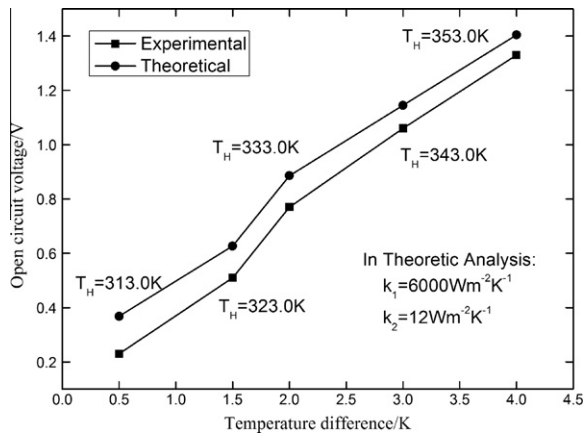


Fig. 4. Effect of  $\Delta T$  on  $U$  characteristic (natural convection cooling).

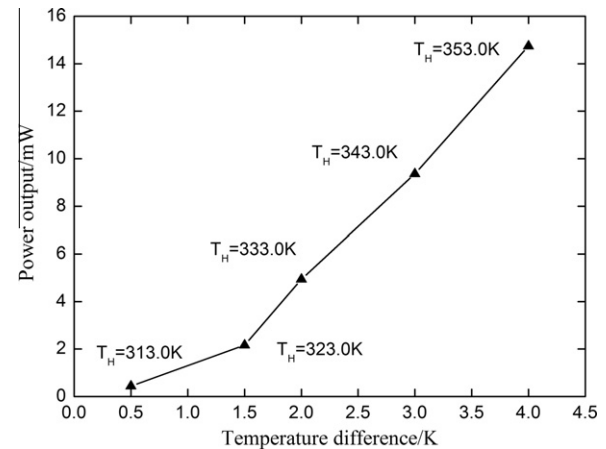


Fig. 6. Effect of  $\Delta T$  on  $P$  characteristic (natural convection cooling).

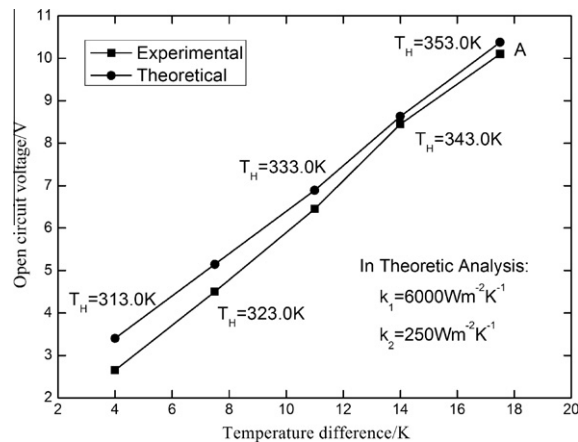


Fig. 5. Effect of  $\Delta T$  on  $U$  characteristic (forced convection cooling).

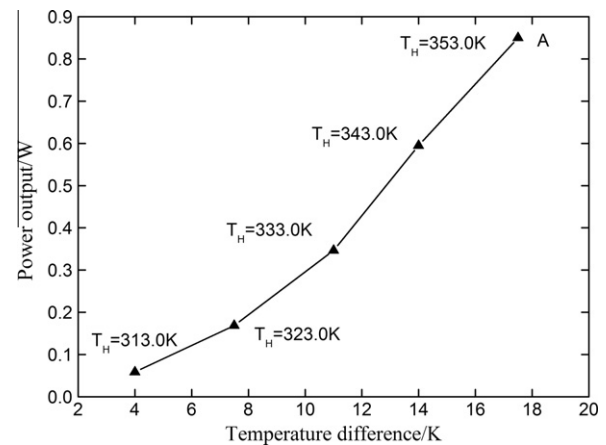


Fig. 7. Effect of  $\Delta T$  on  $P$  characteristic (forced convection cooling).

application of this system, natural and forced convection cooling on cold-side have been particularly compared in this study.

A series of setup open circuit voltage ( $U$ ) values changing with temperature difference have been measured, which are shown in Figs. 4 and 5, by boosting hot water (waste heat) temperature from 313 K to 353 K successively. Fig. 4 is for natural convection cooling, and Fig. 5 is for forced convection cooling (the axial fan running). Temperature difference refers to  $\Delta T$  (multi-point average) between both sides of the TE modules, and  $T_H$  refers to hot water temperature.

Shown in Figs. 4 and 5, system open circuit voltage increases with a boost of temperature difference in a linear trend. What is more, each addition of 10 K to hot water (waste heat) temperature will result in 1 K addition to  $\Delta T$  and 0.3 V addition to  $U$  (open circuit voltage/temperature difference =  $0.3\text{ V K}^{-1}$ ) in natural convection condition, while 3 K addition to  $\Delta T$  and 2 V addition to  $U$  (open circuit voltage/temperature difference =  $0.7\text{ V K}^{-1}$ ) in forced convection condition. System performance has apparently been enhanced by mounting axial fans.

Results of theoretic analysis are shown in Figs. 4 and 5 simultaneously, based on system model established before, with Seebeck coefficient  $\alpha = 2.3 \times 10^{-4}\text{ V K}^{-1}$ , thermal conductance  $K = 1.5 \times 10^{-2}\text{ W K}^{-1}$  and electric resistance  $R = 2.3622 \times 10^{-4}\text{ }\Omega$  [22–24]. It is clearly shown in Figs. 4 and 5 that theoretical curves coincide with experimental curves, whether in the qualitative aspect or the quantitative one. That is to say, reasonability of the system model has been verified, so that we can use this model to guide analysis and optimization of this setup.

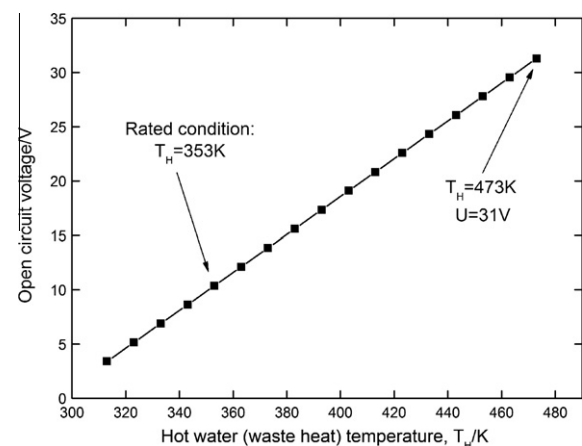


Fig. 8. Effect of  $T_H$  on  $U$  characteristic (theoretic analysis).

According to circuit theory, maximum power output of this system can be attained when road resistance is equal to inner resistance (here,  $30\text{ }\Omega$ ). As a consequence, it is convenient to calculate a series of setup maximum power output ( $P$ ) values changing with temperature difference based on data measured previously (Figs. 6 and 7).

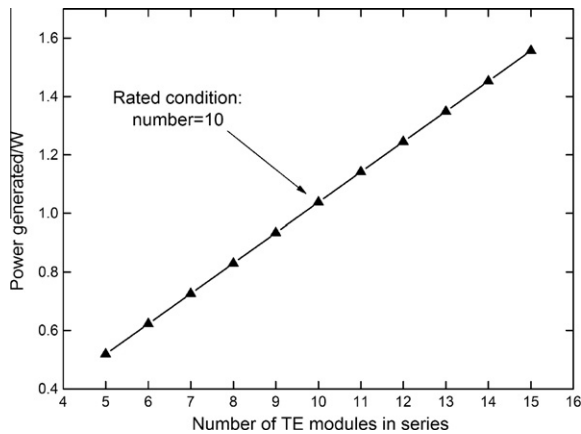


Fig. 9. Effect of TE modules number on  $P$  characteristic (theoretic analysis).

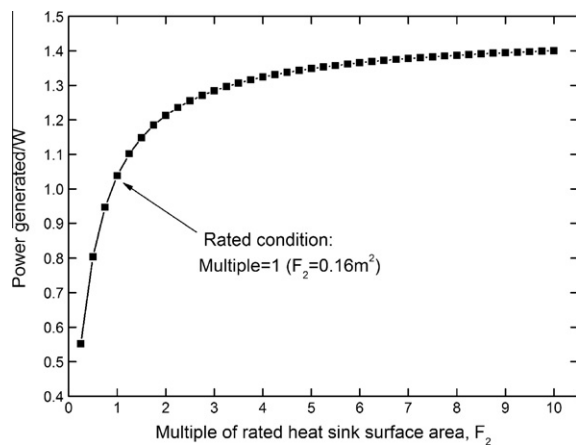


Fig. 10. Effect of  $F_2$  on  $P$  characteristic (theoretic analysis).

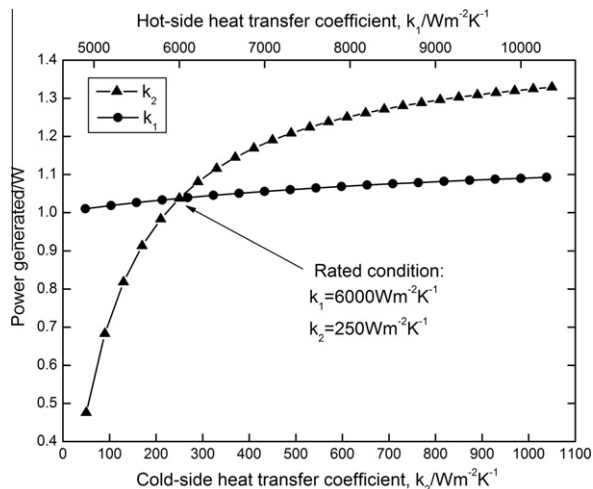


Fig. 11. Comparison concerning the effects of hot- and cold-side heat transfer on  $P$  characteristic (theoretic analysis).

As shown in Figs. 6 and 7, system maximum power output increases while temperature difference is increasing, especially in a quasi-exponential trend. Comparison of power values between Figs. 6 and 7 shows more clearly the effectiveness of employing forced convection cooling.

### 3.3. Discussion

Though the adoption of axial fan can greatly promote system performance, the fan itself is drove by power supply. In this case, a fan need a 6.6 W power to drive, that is to say, this experimental setup is a system unable to run by itself. Fortunately, there are still some methods, such as increasing TE modules in series, expanding heat sink surface area so as to enhance heat transfer capacity on cold-side, which could make the system work so effective that it would self-power or even generate additional power.

In addition, based on Figs. 5 and 7, an exciting analysis can be obtained. In our experimental conditions, cold-side surface temperature has risen only 14 K (room temperature: 24 °C) while hot-side water temperature changing from 313 K to 353 K, namely, cold-side heat dissipation capacity by forced air-cooling is adequate. Hence, in the light of the linear trend of voltage boost (Fig. 5) and the quasi-exponential trend of power increasing (Fig. 7), there would be a temperature difference up to 50 K when waste heat temperature reaches 473 K (200 °C), resulting in a 35 V/10 W around (theoretical result based on system model is 31 V, shown in Fig. 8) power supply which can self-power and even generate additional power. Moreover, this waste heat temperature could be attained in several industrial fields. The promising potential of using thermoelectric generator for low-temperature waste heat recovery, particularly in industrial fields, is demonstrated.

In the aspect of system efficiency, we pay more attention to the thermal efficiency of the TE generator, expressed by Eq. (14), not the whole energy efficiency at present. This is not only because the amount of waste heat in this study is more difficult to measure and evaluate than electric power, but also because the waste heat utilized here is assumed to be no-cost. However, theoretic analysis reveals that the thermal efficiency of this TE generator is only a few percent (2% in Fig. 5 when  $T_H$  reaches 353 K), and the efficiency value dose not change significantly with temperature but increases slightly with a boost of temperature difference.

### 4. System optimization

In order to optimize and modify this setup, the verified system model has been adopted for further analysis on system performance improvement. Here, a rated condition is set as a benchmark for optimization and the rated condition point ( $T_H = 353$  K in forced convection cooling mode) is indicated by "A" in Figs. 5 and 7. Model parameters are the same as that of corresponding theoretic analysis in Figs. 5 and 7.

On the basis of the above discussion, increasing waste heat temperature is a favorable means to promote system performance, especially in industrial fields, which is revealed in Fig. 8. In addition, adding TE modules in series and expanding heat sink surface area are good measures for system improvement. The availability is indicated in Figs. 9 and 10 respectively.

However, the cost of adding TE modules is relatively high and an economic evaluation is needed in almost every project. As shown in Fig. 10, although there is a limit on performance improvement, for this setup, expanding heat sink surface area in a proper range is still an effective method to enhance system performance to some extent.

From the viewpoint of heat transfer, the irreversibility in heat exchange process between TE generator (TE module) and its two heat reservoirs (hot water and ambient air) is an essential condition that confines system performance improvement. In this model, heat transfer coefficients in hot-side ( $k_1$ ) and cold-side ( $k_2$ ) of the generator are two significant parameters, which represent heat transfer capacity and indicate the irreversibility of heat transfer process. Taking the two parameters as independent variables, a



comparison concerning the effects of hot- and cold-side heat transfer on power characteristic is completed (Fig. 11).

As is shown in Fig. 11, for this setup, enhancing cold-side heat transfer capacity in a proper range is an effective way to promote system performance to a certain extent, though there is a limit on power improvement. However, it is not effective to reach the same goal by improving hot-side heat transfer capacity.

## 5. Conclusion

A low-temperature waste heat thermoelectric generator setup has been constructed. At the same time, a theoretic system model based on basic TE effects and the heat transfer irreversibility has also been build. The comparison of results between theoretic analysis and experiment has approved the reasonability of this model. Therefore, this system model can be used in performance optimization, integrated with experiment, and further application of thermoelectric generation.

The promising potential of using thermoelectric generator for low-temperature waste heat recovery, particularly in industrial fields, is demonstrated by the results of theoretic analysis and experiment. Meanwhile, the results show that heat transfer irreversibility does affect thermoelectric generator performance, and irreversibility in cold-side heat transfer of this setup is the bottleneck to enhance system performance.

By means of integrating theoretic analysis and experiment, it is found that besides increasing waste heat temperature and TE modules in series, expanding heat sink surface area in a proper range and enhancing cold-side heat transfer capacity in a proper range can also be employed to enhance performance of this setup.

Further work is planned to conduct a system level optimization study, concerning both energy efficiency and system power capacity, and develop a novel device employing heat pipe technology in a more effective manner for the potential of TE generator waste heat power recovery.

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